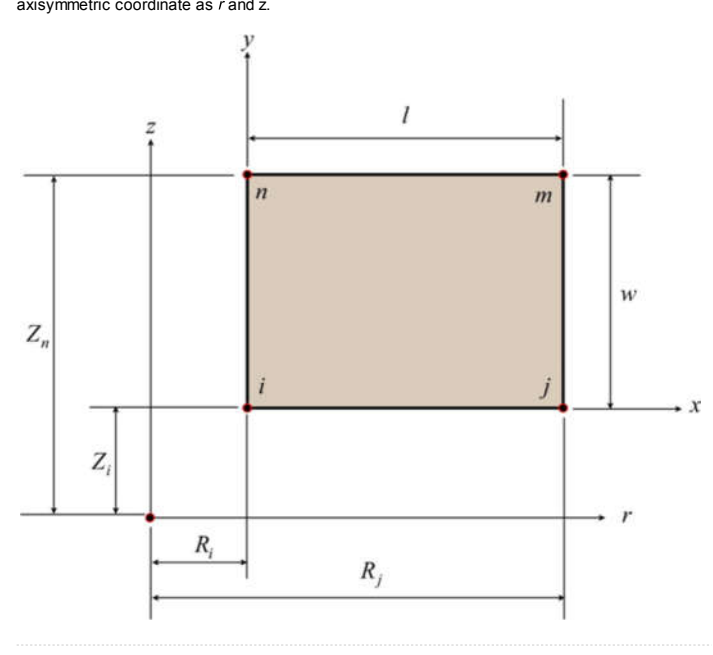
Finite Element Analysis | (4th Edition)

Chapter 7, Problem 25P Bookmark Show all steps: ON

Step-by-step solution

Step 1 of 15

Figure shows an axisymmetric rectangular element with local coordinates as *x* and *y* and the axisymmetric coordinate as *r* and z.



Comment

Step 2 of 15

Consider the axisymmetric rectangular element the shape functions in global coordinate system equation 7.29.

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)$$

Here, R_i, R_j, Z_i, Z_n, l and w are co-ordinate dimensions.

To verify the shape functions of axisymmetric rectangular element, the value of shape function should be unity at the corresponding node and a value of zero at the other nodes.

Comment

Step 3 of 15

Write the following relations from the geometry of figure.

$$Z_n=Z_m$$
$$Z_i=Z_j$$

$$R_i=R_n$$
$$R_j=R_m$$
$$R_j=R_i+l$$
$$l=(R_j-R_i)$$
$$l=(R_m-R_i)\qquad(\because R_j=R_m)$$
$$Z_n=Z_i+w$$
$$w=(Z_n-Z_i)$$
$$w=(Z_n-Z_j)\qquad(\because Z_i=Z_j)$$

Comment

Step 4 of 15

Check the natural coordinates shape function at node *j*.

Consider the natural coordinates shape function at node *j*.

$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)\ \ldots\ldots(1)$$

At node *j*, boundary condition is $r=R_j$ and $z=Z_j$

Substitute R_j for r, and Z_j for z in the equation (1).

$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_j}{w}\right)$$
$$=\left(\frac{l}{l}\right)\left(\frac{w}{w}\right)$$
$$=1$$

Comment

Step 5 of 15

At node *i*, boundary condition is $r=R_i$ and $z=Z_i$

Substitute R_i for r, and Z_i for z in the equation (1).

$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_i-R_i}{l}\right)\left(\frac{Z_n-Z_i}{w}\right)$$
$$=0$$

At node *m*, boundary condition is $r=R_m=R_j$ and $z=Z_m=Z_n$

Substitute R_j for r, and Z_n for z in the equation (1).

$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_n}{w}\right)$$
$$=0$$

Comment

Step 6 of 15

At node *n*, boundary condition is $r=R_n=R_i$ and $z=Z_n$

Substitute R_i for r, and Z_n for z in the equation (1).

$$S_j=\left(\frac{r-R_i}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_i-R_i}{l}\right)\left(\frac{Z_n-Z_n}{w}\right)$$
$$=0$$

Hence, it is proved that shape function is unity at the node *j* and a value of zero at the other nodes.

Comment

Step 7 of 15

Check the natural coordinates shape function at node *m*.

Consider the natural coordinates shape function at node *m*.

$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{z-Z_i}{w}\right)\ \ldots\ldots(2)$$

At node *j*, boundary condition is $r=R_j$ and $z=Z_j=Z_i$

Substitute R_j for r, and Z_i for z in the equation (2).

$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{Z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_i-Z_i}{w}\right)$$
$$=0$$

Comment

Step 8 of 15

At node *i*, boundary condition is $r=R_i$ and $z=Z_i$

Substitute R_i for r, and Z_i for z in the equation (2).

$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_i-R_i}{l}\right)\left(\frac{Z_i-Z_i}{w}\right)$$
$$=0$$

At node *m*, boundary condition is $r=R_m=R_j$ and $z=Z_m=Z_n$

Substitute R_j for r, and Z_n for z in the equation (2).

$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_i}{w}\right)$$
$$=\left(\frac{l}{l}\right)\left(\frac{w}{w}\right)$$
$$=1$$

At node *n*, boundary condition is $r=R_n=R_i$ and $z=Z_n$

Substitute R_i for r, and Z_n for z in the equation (2).

$$S_m=\left(\frac{r-R_i}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_i-R_i}{l}\right)\left(\frac{Z_n-Z_i}{w}\right)$$
$$=0$$

Hence, it is proved that shape function is unity at the node *m* and a value of zero at the other nodes.

Comment

Step 9 of 15

Check the natural coordinates shape function at node *n*.

Consider the natural coordinates shape function at node *n*.

$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)\ \ldots\ldots(3)$$

At node *j*, boundary condition is $r=R_j$ and $z=Z_j=Z_i$

Substitute R_j for r, and Z_i for z in the equation (3).

$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_j}{l}\right)\left(\frac{Z_i-Z_i}{w}\right)$$
$$=0$$

Comment

Step 10 of 15

At node *i*, boundary condition is $r=R_i$ and $z=Z_i$

Substitute R_i for r, and Z_i for z in the equation (3).

$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_i-Z_i}{w}\right)$$
$$=0$$

At node *m*, boundary condition is $r=R_m=R_j$ and $z=Z_m=Z_n$

Substitute R_j for r, and Z_n for z in the equation (3).

$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_j}{l}\right)\left(\frac{Z_n-Z_n}{w}\right)$$
$$=0$$

Comment

Step 11 of 15

At node *n*, boundary condition is $r=R_n=R_i$ and $z=Z_n$

Substitute R_i for r, and Z_n for z in the equation (3).

$$S_n=\left(\frac{R_j-r}{l}\right)\left(\frac{z-Z_i}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_i}{w}\right)$$
$$=\left(\frac{l}{l}\right)\left(\frac{w}{w}\right)$$
$$=1$$

Hence, it is proved that shape function is unity at the node *n* and a value of zero at the other nodes.

Comment

Step 12 of 15

Check the natural coordinates shape function at node *i*.

Consider the natural coordinates shape function at node *i*.

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)\ \ldots\ldots(4)$$

At node *j*, boundary condition is $r=R_j$ and $z=Z_j=Z_i$

Substitute R_j for r, and Z_i for z in the equation (4).

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_j}{l}\right)\left(\frac{Z_n-Z_i}{w}\right)$$
$$=0$$

Comment

Step 13 of 15

At node *i*, boundary condition is $r=R_i$ and $z=Z_i=Z_j$

Substitute R_i for r, and Z_j for z in the equation (4).

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_j}{w}\right)$$
$$=\left(\frac{l}{l}\right)\left(\frac{w}{w}\right)$$
$$=1$$

Comment

Step 14 of 15

At node *m*, boundary condition is $r=R_m=R_j$ and $z=Z_m=Z_n$

Substitute R_j for r, and Z_n for z in the equation (4).

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_j}{l}\right)\left(\frac{Z_n-Z_n}{w}\right)$$
$$=0$$

Comment

Step 15 of 15

At node *n*, boundary condition is $r=R_n=R_i$ and $z=Z_n$

Substitute R_i for r, and Z_n for z in the equation (4).

$$S_i=\left(\frac{R_j-r}{l}\right)\left(\frac{Z_n-z}{w}\right)$$
$$=\left(\frac{R_j-R_i}{l}\right)\left(\frac{Z_n-Z_n}{w}\right)$$
$$=0$$

Hence, it is proved that shape function is unity at the node *i* and a value is zero at the other nodes.

Therefore, the shape functions S_j, S_m, S_n and S_i in the equation 7.29 are verified.

Comment

Was this solution helpful? 0 0