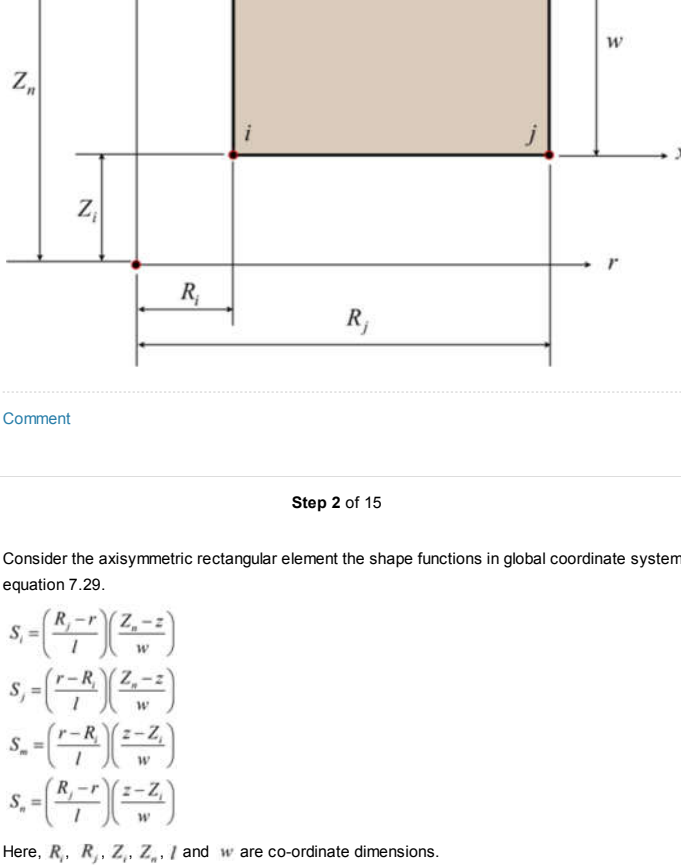


Step-by-step solution

Step 1 of 15

Figure shows an axisymmetric rectangular element with local coordinates as  $x$  and  $y$  and the axisymmetric coordinate as  $r$  and  $z$ .



Comment

Step 2 of 15

Consider the axisymmetric rectangular element shape functions in global coordinate system equation 7.29.

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$S_j = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right)$$

Here,  $R_i$ ,  $R_j$ ,  $Z_i$ ,  $Z_n$ ,  $l$  and  $w$  are co-ordinate dimensions.

To verify the shape functions of axisymmetric rectangular element, the value of shape function should be unity at the corresponding node and a value of zero at the other nodes.

Comment

Step 3 of 15

Write the following relations from the geometry of figure.

$$Z_n = Z_m$$
$$Z_i = Z_j$$
$$R_i = R_n$$
$$R_j = R_m$$
$$R_j = R_i + l$$
$$l = (R_j - R_i) \quad (\because R_j = R_n)$$
$$Z_n = Z_i + w$$
$$w = (Z_n - Z_i) \quad (\because Z_n = Z_j)$$

Comment

Step 4 of 15

Check the natural coordinates shape function at node  $i$ .

Consider the natural coordinates shape function at node  $i$ .

$$S_i = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right) \dots \dots (1)$$

At node  $i$ , boundary condition is  $r = R_i$  and  $z = Z_i$ .

Substitute  $R_i$  for  $r$ , and  $Z_i$  for  $z$  in the equation (1).

$$S_i = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= \left( \frac{l}{l} \right) \left( \frac{w}{w} \right)$$
$$= 1$$

Comment

Step 5 of 15

At node  $i$ , boundary condition is  $r = R_i$  and  $z = Z_i$ .

Substitute  $R_i$  for  $r$ , and  $Z_i$  for  $z$  in the equation (1).

$$S_j = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

At node  $m$ , boundary condition is  $r = R_m = R_j$  and  $z = Z_m = Z_n$ .

Substitute  $R_j$  for  $r$ , and  $Z_n$  for  $z$  in the equation (1).

$$S_j = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_j - R_j}{l} \right) \left( \frac{Z_i - Z_n}{w} \right)$$
$$= 0$$

Comment

Step 6 of 15

At node  $n$ , boundary condition is  $r = R_n = R_i$  and  $z = Z_n$ .

Substitute  $R_i$  for  $r$ , and  $Z_n$  for  $z$  in the equation (1).

$$S_j = \left( \frac{r - R_j}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_n}{w} \right)$$
$$= 0$$

Hence, it is proved that shape function is unity at the node  $j$  and a value of zero at the other nodes.

Comment

Step 7 of 15

Check the natural coordinates shape function at node  $m$ .

Consider the natural coordinates shape function at node  $m$ .

$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right) \dots \dots (2)$$

At node  $j$ , boundary condition is  $r = R_j$  and  $z = Z_j = Z_i$ .

Substitute  $R_j$  for  $r$ , and  $Z_i$  for  $z$  in the equation (2).

$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_j - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

Comment

Step 8 of 15

At node  $i$ , boundary condition is  $r = R_i$  and  $z = Z_i$ .

Substitute  $R_i$  for  $r$ , and  $Z_i$  for  $z$  in the equation (2).

$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

At node  $m$ , boundary condition is  $r = R_m = R_j$  and  $z = Z_m = Z_n$ .

Substitute  $R_j$  for  $r$ , and  $Z_n$  for  $z$  in the equation (2).

$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_j - R_j}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= \left( \frac{l}{l} \right) \left( \frac{w}{w} \right)$$
$$= 1$$

At node  $n$ , boundary condition is  $r = R_n = R_i$  and  $z = Z_n$ .

Substitute  $R_i$  for  $r$ , and  $Z_n$  for  $z$  in the equation (2).

$$S_m = \left( \frac{r - R_j}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= 0$$

Hence, it is proved that shape function is unity at the node  $m$  and a value of zero at the other nodes.

Comment

Step 9 of 15

Check the natural coordinates shape function at node  $n$ .

Consider the natural coordinates shape function at node  $n$ .

$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right) \dots \dots (3)$$

At node  $j$ , boundary condition is  $r = R_j$  and  $z = Z_j = Z_i$ .

Substitute  $R_j$  for  $r$ , and  $Z_i$  for  $z$  in the equation (3).

$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

Comment

Step 10 of 15

At node  $i$ , boundary condition is  $r = R_i$  and  $z = Z_i$ .

Substitute  $R_i$  for  $r$ , and  $Z_i$  for  $z$  in the equation (3).

$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_i}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

At node  $m$ , boundary condition is  $r = R_m = R_j$  and  $z = Z_m = Z_n$ .

Substitute  $R_j$  for  $r$ , and  $Z_n$  for  $z$  in the equation (3).

$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= 0$$

Comment

Step 11 of 15

At node  $n$ , boundary condition is  $r = R_n = R_i$  and  $z = Z_n$ .

Substitute  $R_i$  for  $r$ , and  $Z_n$  for  $z$  in the equation (3).

$$S_n = \left( \frac{R_i - r}{l} \right) \left( \frac{z - Z_i}{w} \right)$$
$$= \left( \frac{R_i - R_i}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= \left( \frac{l}{l} \right) \left( \frac{w}{w} \right)$$
$$= 1$$

Hence, it is proved that shape function is unity at the node  $n$  and a value of zero at the other nodes.

Comment

Step 12 of 15

Check the natural coordinates shape function at node  $i$ .

Consider the natural coordinates shape function at node  $i$ .

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right) \dots \dots (4)$$

At node  $j$ , boundary condition is  $r = R_j$  and  $z = Z_j = Z_i$ .

Substitute  $R_j$  for  $r$ , and  $Z_i$  for  $z$  in the equation (4).

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= 0$$

Comment

Step 13 of 15

At node  $i$ , boundary condition is  $r = R_i$  and  $z = Z_i$ .

Substitute  $R_i$  for  $r$ , and  $Z_i$  for  $z$  in the equation (4).

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_i}{l} \right) \left( \frac{Z_i - Z_i}{w} \right)$$
$$= \left( \frac{l}{l} \right) \left( \frac{w}{w} \right)$$
$$= 1$$

Comment

Step 14 of 15

At node  $m$ , boundary condition is  $r = R_m = R_j$  and  $z = Z_m = Z_n$ .

Substitute  $R_j$  for  $r$ , and  $Z_n$  for  $z$  in the equation (4).

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_j}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= 0$$

Comment

Step 15 of 15

At node  $n$ , boundary condition is  $r = R_n = R_i$  and  $z = Z_n$ .

Substitute  $R_i$  for  $r$ , and  $Z_n$  for  $z$  in the equation (4).

$$S_i = \left( \frac{R_i - r}{l} \right) \left( \frac{Z_i - z}{w} \right)$$
$$= \left( \frac{R_i - R_i}{l} \right) \left( \frac{Z_n - Z_i}{w} \right)$$
$$= 0$$

Hence, it is proved that shape function is unity at the node  $i$  and a value is zero at the other nodes.

Therefore, the shape functions  $S_i, S_j, S_m, S_n$  and  $S_i$  in the equation 7.29 are verified.

Comment

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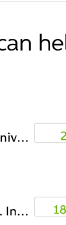
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